A TRACTABLE FRAMEWORK TO RELATE MARGINAL WILLINGNESS-TO-PAY IN HEDONIC AND DISCRETE CHOICE MODELS

Abstract

The two primary approaches to estimate marginal willingness-to-pay (MWTP) are hedonic (Rosen, 1974) and discrete choice models (McFadden, 1974). This paper provides a tractable framework to investigate the relationship between MWTP in these models. By deriving the hedonic price gradient implicitly from the share function in the discrete choice model, I present an analytical mapping between the hedonic gradient (hence, the hedonic MWTP) and choice probabilities in the discrete choice model. Intuitively, the hedonic MWTP depends on *weighted averages* of marginal utilities where higher weights are assigned to individuals whose choice probabilities indicate more uncertain choices (*marginal* individuals). As this choice becomes more certain, the weights start to decrease. Since the hedonic method relies on tangencies between indifference curves and the hedonic price function to identify MWTP, inframarginal individuals (whose bid functions are not tangent to the hedonic price function) have low weights (they have choice probabilities that are close to 0 or 1 and low choice variances). This novel analytical mapping between the hedonic gradient and the share function can be used to identify conditions when MWTP in the two models are similar. (JEL: C01, R21, J23)[1]

Keywords: Hedonic, discrete choice, sorting, demand

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1 Introduction

The two primary approaches to estimate marginal willingness-to-pay (MWTP) are the hedonic (Rosen, 1974) and discrete choice models (McFadden, 1974). While both approaches are used widely in many fields,[2] there is little formal analysis of the relationship between both models. Moreover, some papers that use both approaches to estimate MWTP find different results but it is hard to investigate why the estimates differ without a framework that directly relates both models. For example, Banzhaf (2002) finds that the MWTP for the same change in air quality varies between $8 (hedonic) to $18-$25 (discrete choice) using the same data.

This paper provides a tractable framework to investigate the relationship between MWTP in the hedonic and discrete choice models. I focus on the housing context where both approaches are used widely. To establish this relationship, I begin with a discrete choice model of heterogeneous individuals choosing houses to maximize their utility. I use a standard utility function with random coefficients and an idiosyncratic Logit error term. Houses are differentiated by a vector of characteristics, including price. An equilibrium is characterized by a vector of prices and an allocation of individuals to houses such that each individual has no incentive to deviate. Notably, in the discrete choice framework, choices made by individuals can be summarized by choice probabilities using a *share function* that indicates the share of individuals in a market who choose a house as a function of house characteristics and house prices.

The hedonic approach presents a dual way to characterize equilibria in markets with differentiated choices using the *hedonic price function*.[3] Rosen (1974) showed that a utility-maximizing individual choosing amongst a continuum of differentiated houses satisfies the first order conditions of optimization when his indifference curve is tangent to the hedonic price function. This is the famous insight in Rosen (1974) that an individual's MWTP for a characteristic is equal to the gradient of the hedonic price function with respect to that characteristic. Section 2 describes both approaches and highlights the hedonic price function and the share function as key objects of interest in the hedonic and discrete choice models, respectively.

Section 3 investigates the relationship between the hedonic price function and the share function and delivers three results. The first result is an analytical mapping that relates the gradient of the hedonic price function to the share function in a discrete choice model. Given a discrete choice model (as described in Section 2.1), the key insight is to derive the gradient of the hedonic price function implicitly from the share function of this discrete choice model using the Implicit

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[2]See Bayer et al. (2007); Cellini et al. (2008); Chay and Greenstone (2004); Pakes (2003); Berry et al. (1995); Bitzan and Wilson (2007); Wong (2013), to name a few examples in the fields of labor economics, local public finance, environmental economics, industrial organization as well as urban and transportation economics.

[3]The hedonic price function maps characteristics of houses to prices of houses.

Function Theorem. This relates the hedonic gradient (hence, MWTP in the hedonic model) with choice probabilities and the share function in the discrete choice model.

The second result is that MWTP in the hedonic model can be expressed as a ratio of weighted averages of individual marginal utilities. The weights are a function of choice probabilities in the discrete choice model with higher weights corresponding to individuals with more uncertain choices. This result relies on the assumption that the hedonic price function is only a function of own-house characteristics. This is a common assumption in the empirical literature. For example, in the housing literature, the hedonic price function is often estimated by regressing the price of a house on the characteristics of that house (but not on the characteristics of other houses in the market).

To interpret the economic intuition behind these probability weights, the key observation is that the hedonic method relies on tangencies between the indifference curves and the hedonic price function to identify MWTP using the hedonic gradient. However, with heterogeneous individuals, only the *marginal* individuals' indifference curves are tangent to the hedonic price function. The indifference curves of *inframarginal* individuals are not necessarily tangent to the hedonic price function.

To give an example that shows how to use probability weights to distinguish marginal and inframarginal individuals, consider an individual whose probability of choosing a house is one. I find that the hedonic estimate of MWTP assigns no weight to this individual. This is because he chooses a house with certainty (he is inframarginal and his indifference curve is not tangent to the hedonic price function). More generally, I find that the hedonic MWTP depends on a ratio of weighted averages of marginal utilities where higher weights are assigned to individuals whose choice probabilities indicate a higher degree of uncertainty regarding their choice (marginal individuals). As this choice becomes more certain (as the probability approaches 0 or 1), the weights start to decrease.

The second result implies that we can use other moments from choice data (choice variance and choice probability) to determine which individuals are marginal versus inframarginal. Since the hedonic model assumes a continuum of houses, in principle, each individual is marginal because he can always find a house where his indifference curve is tangent to the hedonic price function. Therefore, the theory does not provide guidance on how to determine which types of individuals are more likely to be marginal. The analytical relationship between the share function and the hedonic gradient provides a theoretical justification for using choice data to identify marginal individuals. This complements the hedonic approach which typically only utilizes data on prices but not choice data (quantities).

Aside from this, the second result also provides a tractable way to identify conditions under which

MWTP from both models are identical. It shows clearly that MWTP in the hedonic model depends on a ratio of (weighted) averages of marginal utilities whilst MWTP in the discrete choice model is an average of ratios of marginal utilities. Generally, the ratio of averages will not equal the average of ratios except in special cases (for example, when the ratios are constant).

This intuition delivers the third finding that the average MWTP for a characteristic is identical in the two models if the MWTP for that characteristic is constant across individuals. The traditional Logit model with no random coefficients satisfies this condition. This appears to be a special case when ratios of marginal utilities (MWTP's) are constant. With heterogeneous preferences for characteristics (for example, with random coefficients utility), some individuals could be marginal and others could be inframarginal. Only the slopes of the indifference curves of *marginal* individuals are equal to the hedonic gradient. Therefore, the average MWTP in the hedonic model (which gives higher weights to marginal individuals) diverges from the average MWTP in the discrete choice model (which estimates an average MWTP, averaged across marginal and inframarginal individuals). In contrast, when MWTP for a characteristic is constant, the marginal individual and the average individual have the same MWTP, so, average MWTP estimated in the two models are the same.

While there are many empirical papers that use the hedonic and discrete choice methods to estimate MWTP, there is a relatively small literature that directly compares estimates from both models. Cropper et al. (1993) and Mason and Quigley (1990) use simulated data to compare MWTP estimates in hedonic and discrete choice models. Several papers allude to similarities and differences between both models (Ellickson, 1981; Ekeland et al., 2004; Bayer et al., 2007; Bajari and Benkard, 2005). The innovation in this paper is to provide a tractable framework that delivers an analytical relationship between the gradient of the hedonic price function and the share function in the discrete choice model.

The remainder of the paper is organized as follows: I briefly describe the discrete choice and hedonic models in Section 2. I derive the three results above in Section 3 and discuss their implications. Finally, I conclude in Section 4.

2   Discrete choice and hedonic models

The goal is to provide an analytical mapping that relates MWTP in the hedonic model with MWTP in the discrete choice model. The theoretical exercise starts with a discrete choice model as the underlying data generating process and describes an equilibrium in this model, given a fixed supply. I use a standard discrete choice model with functional form and distributional assumptions that are

commonly used in the empirical literature. Then, I describe how an equilibrium is analogously characterized in the hedonic model. This discussion highlights two objects of interest: the share function (in the discrete choice framework) and the hedonic price function (in the hedonic framework). The next section derives an analytical mapping between the share function and the hedonic price function.

Throughout this paper, I take the supply side as given and focus mainly on describing consumer preferences and the demand side. The results in this paper are derived holding the functional forms and distributional assumptions described in Section 2.1 fixed. I discuss how my approach can be generalized to other settings later.

2.1 Discrete choice model and the share function

There are $t = 1, ..., T$ markets and each market has J_t differentiated houses. Individual i's indirect utility from choosing house j in market t is,

$$u_{ijt} = V(x_{jt}, p_{jt}; \beta_i, y_i) + \varepsilon_{ijt} \tag{1}$$

where y_i is the income of individual i, p_{jt} is the price of house j in market t, x_{jt} is a K-dimensional (row) vector of exogenous characteristics of house j.[4] The numeraire good, $y_i - p_{jt}$, has a normalized price of 1. Each individual i has heterogeneous taste parameters for house characteristics (β_i drawn from a cumulative distribution function, F_β) and a random taste parameter for house j (ε_{ijt} drawn from F_ε). The model is closed with an outside good, $j = 0$. The utility from the outside good is normalized to 0. Each market is independent from other markets. To simplify notation, I will drop the market subscript from here.

The empirical literature makes two common assumptions for equation (1). First, $V(x_{jt}, p_{jt}; \beta_i, y_i)$ is a random coefficients utility function. Second, ε is drawn from a Type I extreme value distribution. For example, a common functional form is

$$u_{ij} = \beta_{ip}(y_i - p_j) + x_j \beta_i + \varepsilon_{ij} \tag{2}$$

where β_{ip} is the marginal utility of income.[5] In this discrete choice model, the MWTP of individual i for characteristic k is

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[4]For simplicity, this model assumes all characteristics other than price are exogenous. It also assumes no observed heterogeneity amongst individuals. Both assumptions can easily be relaxed.

[5]This model assumes quasi-linear utility. One can also model piecewise income effects (Petrin, 2002) or Cobb-Douglas utility functions (Berry et al., 1995). Some models also assume an unobserved quality term for house j, ξ_j. The key intuition will follow through with these modifications to the utility function.

$$MWTP_{ik}^D = \frac{\beta_{ik}}{\beta_{ip}} \tag{3}$$

and the average MWTP for characteristic k is $MWTP_k^D = \int \frac{\beta_{ik}}{\beta_{ip}} dF_\beta$. An individual chooses house j that offers the highest utility. Let A_j be the set of individuals who choose j:

$$A_j = \left\{ \left(\beta_i, \beta_{ip}, \varepsilon_{i0}, \varepsilon_{i1}, ..., \varepsilon_{iJ} | u_{ij} \geq u_{ik}, k = 0, ..., J \right) \right\} \tag{4}$$

The share of individuals in a market who choose house j (π_j) is obtained from aggregating across individuals in A_j,

$$\begin{aligned} \pi_j (\mathbf{x}, \mathbf{p}) &= \int_{A_j} dF_\beta dF_\varepsilon \\ &= \int \frac{exp(V_{ij})}{\sum_{j'=0}^{j'=J} exp(V_{ij'})} dF_\beta \equiv \int \pi_{ij} dF_\beta \end{aligned} \tag{5}$$

where the second row shows that the probability that i chooses j (π_{ij}) is $\frac{exp(V_{ij})}{\sum_{j'=0}^{j'=J} exp(V_{ij'})}$ because the $\varepsilon's$ are drawn from a Type I extreme value distribution.

An equilibrium is characterized by a vector of prices for each house and an allocation of individuals to houses so that no one has an incentive to deviate. The *share function*, $\pi(.)$, can be used to concisely summarize the optimizing choices individuals make in the discrete choice model. Given a fixed supply, in equilibrium, each element (π_j^*) in the J-dimensional vector (π^*) summarizes the share of individuals in a market who choose house j, as a function of house characteristics and equilibrium prices, evaluated at $(\mathbf{x}, \mathbf{p}^*)$.

2.2 Hedonic model and the hedonic price function

The hedonic model offers a dual way to describe an equilibrium in a housing market. Each market has a continuum of houses. A house is differentiated along a bundle of characteristics, $\mathbf{x}$. As in the discrete choice framework above, I assume supply is fixed. Individual i takes the market price for houses, $P(\mathbf{x})$, as given and chooses one unit of a house to maximize utility, u_i, subject to the budget constraint, $P(\mathbf{x}) + numeraire \leq y_i$. The numeraire is normalized to have a price of 1..

Given supply, an equilibrium in the hedonic model is characterized by individuals who are maximizing utilities given their budget constraints. Graphically, individual i's taste for x_k can be illustrated using bid functions (indifference curves in $P - x_k$ space) with steeper bid functions representing stronger taste (higher MWTP) for x_k. Each individual chooses a house that corresponds to

the bid function that maximizes his utility. Under the first order conditions, optimality is achieved when the MWTP for x_k (the ratio of the marginal utility for x_k and the marginal utility for the numeraire) is equal to the ratio of the marginal cost for x_k and the marginal cost for the numeraire (normalized to 1). In the hedonic model, individual $i's$ MWTP for characteristic k is

$$MWTP_{ik}^H = \frac{\partial u_i/\partial x_k}{\partial u_i/\partial P} = \frac{\partial P}{\partial x_k} \tag{6}$$

Prices adjust so that each house is sold to the highest bidder and the marginal individual is just indifferent between a marginal gain in utility from choosing an additional unit of x_k and incurring a marginal cost for it (relative to the numeraire). Equilibrium interactions in the market trace out a price-characteristic $(P - x_k)$ locus that implicitly defines a market clearing, *hedonic price function*, $P(\mathbf{x})$. The hedonic price function is the upper envelop of bid functions. Importantly, equation (6) delivers the famous insight from Rosen (1974) that the gradient of the hedonic price function (with respect to x_k) is equal to individual $i's$ MWTP for x_k (the bid function for individual i is tangent to the hedonic price function).

3 Results

This section builds on the discrete choice and hedonic models described in Section 2 to provide a tractable framework to compare MWTP from both approaches. The analysis delivers the three results below. The first result is an analytical mapping between the hedonic price gradient and the share function.

Result 1: *In the discrete choice model described in Section 2.1, if an equilibrium exists that can be represented by a hedonic price function as described in Section 2.2, then, the gradient of the hedonic price function can be written as a function of choice probabilities using the share function in the discrete choice model.*

This result is an application of the Implicit Function Theorem (Theorem 15.7 in Simon and Blume (1994)). Let $\pi_1, ..., \pi_J : \mathbf{R^{J(K+1)}} \to \mathbf{R^1}$ be C^1 functions. Consider a system of J equations

$$
\begin{aligned}
\pi_1(p_1, ..., p_J, x_{11}, ..., x_{1K}, ..., x_{jk}, ..., x_{J1}, ..., x_{JK}) &= \pi_1^* \\
&\vdots \\
&\vdots \\
\pi_J(p_1, ..., p_J, x_{11}, ..., x_{1K}, ..., x_{jk}, ..., x_{J1}, ..., x_{JK}) &= \pi_J^*
\end{aligned} \tag{7}
$$

as possibly defining $p_1, ..., p_J$ as implicit functions of $x_{11}, ..., x_{JK}$. The left hand side of each equa-

tion j is the share function for house j and the right hand side is the share of individuals in the market choosing that house. Suppose that $(\mathbf{p^*}, \mathbf{x})$ is a solution of (7). If the determinant of the $J \times J$ matrix

$$
\begin{bmatrix}
\frac{\partial \pi_1}{\partial p_1} & \cdots & \frac{\partial \pi_1}{\partial p_J} \\
\vdots & \ddots & \vdots \\
\frac{\partial \pi_J}{\partial p_1} & \cdots & \frac{\partial \pi_J}{\partial p_J}
\end{bmatrix}
$$

evaluated at $(\mathbf{p^*}, \mathbf{x})$ is nonzero (ie. the matrix is invertible), then there exist C^1 functions in $\mathbf{R}^{J(K+1)}$

$$
P_1(x_{11},...,x_{JK}) = p_1
$$

(8)

$$
P_J(x_{11},...,x_{JK}) = p_J
$$

defined on a ball B about $\mathbf{x}$ such that

$$
\pi_1(P_1(\mathbf{x}),...,P_J(\mathbf{x}),x_{11},...,x_{1K},...x_{jk},...,x_{J1},...,x_{JK}) = \pi_1^*
$$

$$
\vdots
$$

$$
\pi_J(P_1(\mathbf{x}),...,P_J(\mathbf{x}),x_{11},...,x_{1K},...x_{jk},...,x_{J1},...,x_{JK}) = \pi_J^* \qquad (9)
$$

for all $\mathbf{x} = (x_{11},...,x_{JK})$ in B and the gradient of this implicit function is

$$
\begin{bmatrix}
\partial P_1 / \partial x_{jk} \\
\vdots \\
\partial P_J / \partial x_{jk}
\end{bmatrix}
= -
\begin{bmatrix}
\frac{\partial \pi_1}{\partial p_1} & \cdots & \frac{\partial \pi_1}{\partial p_J} \\
\vdots & \ddots & \vdots \\
\frac{\partial \pi_J}{\partial p_1} & \cdots & \frac{\partial \pi_J}{\partial p_J}
\end{bmatrix}^{-1}
\begin{bmatrix}
\partial \pi_1 / \partial x_{jk} \\
\vdots \\
\partial \pi_J / \partial x_{jk}
\end{bmatrix}
\qquad (10)
$$

Since ε is Type I extreme value and independent from F_β, we know from (5) that $\pi_j = \int \frac{exp(V_{ij})}{\sum_{j'=0}^{j'=J} exp(V_{ij'})} dF_\beta$. The partial derivatives on the right hand side are:

$$
\frac{\partial \pi_j}{\partial x_{jk}} =
\begin{cases}
\int \frac{\partial V_{ij}}{\partial x_{jk}} \pi_{ij}(1 - \pi_{ij}) dF_\beta & \\
\int \frac{\partial V_{ij}}{\partial x_{j'k}} \pi_{ij} \pi_{ij'} dF_\beta & j \neq j'
\end{cases}
\qquad \text{and} \qquad
\frac{\partial \pi_j}{\partial P_j} =
\begin{cases}
\int \frac{\partial V_{ij}}{\partial P_j} \pi_{ij}(1 - \pi_{ij}) dF_\beta & \\
\int \frac{\partial V_{ij}}{\partial P_{j'}} \pi_{ij} \pi_{ij'} dF_\beta & j \neq j'
\end{cases}
$$

Furthermore, if V_{ij} has random coefficients utility (2), then, $\frac{\partial V_{ij}}{\partial x_{jk}} = \frac{\partial V_{ij}}{\partial x_{j'k}} = \beta_{ik}$ and $\frac{\partial V_{ij}}{\partial P_j} = \frac{\partial V_{ij}}{\partial P_{j'}} = \beta_{ip}$.

This result delivers an analytical relationship between the gradient of the hedonic price function and the share function in the discrete choice model. The steps from (7) to (9) use the share function, $\pi(\mathbf{x}, \mathbf{p})$, to implicitly define price as a function of $\mathbf{x}$, $P(\mathbf{x})$. Then, (10) relates the gradient of the (implicitly defined) hedonic price function to changes in the share function. Locally around the

equilibrium point, $(\mathbf{p^*}, \mathbf{x})$, a small change in x_{jk} will induce individuals' choices to change, which in turn, leads to changes in market shares. Therefore, the hedonic (implicit) price of an additional unit of x_{jk} ($\partial \mathbf{P}/\partial x_{jk}$) should depend both on the impact of a small change in x_{jk} on market shares as well as the impact on market shares when prices change (the terms on the right of equation(10)).

The analytical relationship in (10) is useful because it represents a mapping between the gradient of the hedonic price function (hence, MWTP in the hedonic model) and the share function in the discrete choice model. This mapping can be used to identify conditions under which MWTP estimates in both approaches will be similar, which we turn to next.

Result 2: *If the hedonic price function is a function of own-house characteristics only, then, MWTP in the hedonic model can be written as a ratio of weighted averages of marginal utilities where the weights depend on choice probabilities in the discrete choice model.*

While the analytical relationship in (10) is useful, it is still hard to interpret because it is complicated by the inverse of the *JxJ* matrix in (10). The second result shows that this relationship can be simplified if the hedonic price function, $P(\mathbf{x})$, is only a function of own-house characteristics, so that $\frac{\partial P_j}{\partial x_{j'k}} = 0$ for $j \neq j'$.[6] This assumption reduces the dimensionality of the hedonic price function from $\mathbf{R^{J(K+1)}}$ to $\mathbf{R^{(K+1)}}$. It is a common assumption made in the empirical literature. For example, in the housing literature, the hedonic price function is typically estimated by regressing the price of a house on the characteristics of that house only (but rarely on the characteristics of other houses).

To derive the second result, differentiate each row j of (9) with respect to x_{jk},

$$\frac{\partial \pi_1}{\partial P_1} \frac{\partial P_1}{\partial x_{1k}} = -\frac{\partial \pi_1}{\partial x_{1k}}$$

$$\vdots$$

$$\frac{\partial \pi_J}{\partial P_J} \frac{\partial P_J}{\partial x_{Jk}} = -\frac{\partial \pi_J}{\partial x_{Jk}} \tag{11}$$

where the additional terms on the left hand side of (11) are 0 because $\frac{\partial P_j}{\partial x_{j'k}} = 0$. Therefore, we can re-write (11) as

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[6]For example, if k represents the square footage of a house, this assumption states that locally around the equilibrium, the price of house j depends on its square footage but the square footage of other houses will not affect the price of house j.

$$
\begin{bmatrix} \partial P_1/\partial x_{1k} \\ \vdots \\ \partial P_J/\partial x_{jk} \end{bmatrix} = - \begin{bmatrix} \frac{\partial \pi_1/\partial x_{1k}}{\partial \pi_1/\partial P_1} \\ \vdots \\ \frac{\partial \pi_J/\partial x_{Jk}}{\partial \pi_J/\partial P_J} \end{bmatrix} \tag{12}
$$

where $\frac{\partial \pi_j}{\partial x_{jk}} = \int \beta_{ik}\pi_{ij}(1-\pi_{ij})dF_\beta$ and $\frac{\partial \pi_j}{\partial P_j} = \int \beta_{ip}\pi_{ij}(1-\pi_{ij})dF_\beta$.

Equation (12) indicates that the gradient of the hedonic price function can be written as a ratio of weighted averages of marginal utilities ($\partial P_j/\partial x_{jk} = \frac{\int w_{ij}\beta_{ik}dF_\beta}{\int w_{ij}\beta_{ip}dF_\beta}$), where the weights, w_{ij}, are a function of choice probabilities in the discrete choice model ($w_{ij} = \pi_{ij}(1-\pi_{ij})$). These weights represent the variance of individual $i's$ choices. Equation (12) gives 0 weight to individuals whose choice probabilities are 1 or 0. This is because these are individuals who will choose (not choose) a house with certainty (the variance of their choice is 0). Conversely, equation (12) gives the maximum weight to individuals whose choice probability is 0.5.[7] These are individuals who have the highest choice variance and are on the margin of choosing or not choosing a house.

The key insight is that the hedonic method relies on the tangency between the bid functions and the hedonic price function (see first order conditions in (6)) but only the marginal individual's bid function is tangent to the hedonic price function. Therefore, the hedonic method gives a higher weight to marginal individuals whose choice probabilities indicate a higher degree of uncertainty regarding their choices. As this choice becomes more certain (π_{ij} approaching 0 or 1), the weights decrease.

The weights imply a divergence between MWTP in the two approaches because the hedonic approach assumes a continuum of houses whereas the discrete choice model does not. Since housing markets are thin, with discrete houses and individuals with heterogeneous tastes, some individuals may be inframarginal. For example, given a (discrete) set of houses in the market, the utility-maximizing choice for an individual could be where $\frac{\partial u_i/\partial x_k}{\partial u_i/\partial P} > \frac{\partial P}{\partial x_k}$, so that this individual would prefer to pay $\frac{\partial P}{\partial x_k}$ for an additional unit of characteristic k (for example, square footage) but there is no such house available. The discreteness could arise simply due to frictions such as fixed costs of production or other frictions that give rise to thin housing markets (Arnott, 1989; Gavazza, 2011). Importantly, the bid function that maximizes utility for an inframarginal individual is not tangent

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[7]The max at 0.5 is a consequence of the Type I extreme value assumption. This distributional assumption implies that the choice probabilities are drawn from a logistic distribution. This is because choices are driven by differences in random utilities and the difference between two random variables of Type I extreme value distribution is a random variable drawn from the logistic distribution. Logistic distributions have a cumulative distribution function that is sigmoid shape with the steepest slope at 0.5.

to the hedonic price function. Therefore, the gradient of the hedonic price function cannot identify the MWTP for inframarginal individuals.

In theory, with a continuum of houses, the hedonic model assumes everyone is marginal in that the first order conditions of every individual i satisfies the tangency condition in (6). So, the theory does not provide guidance on how to identify marginal versus inframarginal individuals. Moreover, most hedonic applications only utilize data on prices but not data on quantities and choices because of a common assumption that each individual chooses one unit of housing. Result 2 uses the analytical relationship in (12) to provide a theoretical justification for using higher moments of choice probabilities (choice variance) to determine which individuals are marginal versus inframarginal.

Result 3: *The average MWTP for characteristic k from the discrete choice model is equal to the average MWTP for characteristic k from the hedonic model if the ratios of marginal utilities ($\frac{\beta_{ik}}{\beta_{ip}}$) are constant across all individuals. The traditional Logit model with no random coefficients satisfies this condition.*

This result compares the average MWTP for characteristic k estimated using the hedonic approach to the average MWTP in the discrete choice model, averaged across $MWTP_{ik}^D$ and $MWTP_{ik}^H$, as defined in (3) and (6), respectively

$$MWTP_k^D = \int \frac{\beta_{ik}}{\beta_{ip}} dF_\beta \tag{13}$$

$$MWTP_k^H = \frac{1}{J} \sum_j \partial P_j / \partial x_{jk} = \frac{1}{J} \sum_j \frac{\int w_{ij} \beta_{ik} dF_\beta}{\int w_{ij} \beta_{ip} dF_\beta} \tag{14}$$

since the average MWTP estimated in the hedonic model is the average of the slope of the hedonic price function and $\partial P_j / \partial x_{jk} = \frac{\int w_{ij} \beta_{ik} dF_\beta}{\int w_{ij} \beta_{ip} dF_\beta}$ from equation (12).

Generally, the two estimates of average MWTP will be different because the discrete choice method estimates the average of ratios ($\int \frac{\beta_{ik}}{\beta_{ip}} dF_\beta$) and the hedonic estimate depends on the ratio of (weighted) averages. For $MWTP_k^D = MWTP_k^H$, we need the ratios ($\frac{\beta_{ik}}{\beta_{ip}}$) to be constant across all $i's$ so that the average of ratios equals the ratio of averages. If $\beta_{ik} = c\beta_{ip}$ for all i and for some constant c, then, $MWTP_k^D = \int \frac{\beta_{ik}}{\beta_{ip}} dF_\beta = c$ and $\partial P_j / \partial x_{jk} = \frac{\int w_{ij} \beta_{ik} dF_\beta}{\int w_{ij} \beta_{ip} dF_\beta} = \frac{\int c w_{ij} \beta_{ip} dF_\beta}{\int w_{ij} \beta_{ip} dF_\beta} = c$. So, $MWTP_k^H = c$ also.

The traditional Logit model with no random coefficients satisfies this condition. Without random coefficients, equation (2) reduces to $u_{ij} = \bar{\beta}_p (y_i - p_j) + x_j \bar{\beta} + \varepsilon_{ij} = V_j + \varepsilon_{ij}$. So, $MWTP_k^D =$

$\int \frac{\beta_{ik}}{\beta_{ip}} dF_\beta = \frac{\bar{\beta}_k}{\bar{\beta}_p}$. Also, the share function simplifies from $\pi_j = \int \frac{exp(V_{ij})}{\sum_{j'=0}^{j'=J} exp(V_{ij'})} dF_\beta$ to $\frac{exp(V_j)}{\sum_{j'=0}^{j'=J} exp(V_{j'})}$.

Therefore, applying (12) to the simplified share function, $\frac{\partial \pi_j}{\partial x_{jk}} = \bar{\beta}_k \pi_j (1 - \pi_j)$ and $\frac{\partial \pi_j}{\partial P_j} = \bar{\beta}_p \pi_j (1 - \pi_j)$. And, $\frac{\partial P_j}{\partial x_{jk}} = -\frac{\partial \pi_j / \partial x_{jk}}{\partial \pi_j / \partial P_j} = \frac{\bar{\beta}_k \pi_j (1 - \pi_j)}{\bar{\beta}_p \pi_j (1 - \pi_j)} = \frac{\bar{\beta}_k}{\bar{\beta}_p}$ for all j. Therefore, $MWTP_k^D = MWTP_k^H = \frac{\bar{\beta}_k}{\bar{\beta}_p}$.

Intuitively, without random coefficients, there is no heterogeneity in the taste for house characteristics and only heterogeneity in the taste for houses (ε_{ij}). So, $MWTP_{ik}$ is constant across individuals ($\frac{\beta_{ik}}{\beta_{ip}} = \frac{\bar{\beta}_k}{\bar{\beta}_p}$). Individuals have bid functions with identical slopes with respect to k (this is akin to having a representative consumer). Accordingly, the hedonic price function has a constant gradient with respect to k equal to the constant, $\frac{\bar{\beta}_k}{\bar{\beta}_p}$. In other words, the representative consumer is also the average consumer *and* the marginal consumer so there is no wedge between $MWTP_k^D$ which estimates the MWTP for the average consumer and $MWTP_k^H$ gives higher weights to marginal consumers.

4 Conclusion

Marginal willingness-to-pay (MWTP) is important for welfare analysis. The two primary approaches to estimate MWTP are hedonic (Rosen, 1974) and discrete choice models (McFadden, 1974). The innovation in this paper is to provide a tractable framework that delivers a novel analytical mapping between MWTP in the hedonic model and MWTP in the discrete choice model. My analysis delivers three results.

First, we can use the share function in the discrete choice model to define the gradient of the hedonic price function implicitly. Second, if we further assume that the hedonic price function is only a function of own-attributes (a common assumption in the empirical literature), then, the gradient of the hedonic price function depends on a ratio of weighted averages of marginal utilities with higher weights for individuals with more uncertainty in choices (marginal individuals). Third, the average MWTP in both models are identical if MWTP is constant across individuals. The traditional Logit model without random coefficients satisfies this condition.

The framework presented here resonates with previous work that have alluded to the duality between both the hedonic and discrete choice approaches. To my knowledge, this paper provides the first tractable framework that directly links MWTP from both approaches. The analytical mapping is transparent and details some assumptions under which the MWTP from both approaches will be identical. Moreover, the framework illustrates why MWTP can differ and how one can use choice data to examine which types of individuals are more likely to be marginal, thereby complementing

the hedonic approach that typically uses price data only. Intuitively, the differences will depend on the context and discreteness in the choice sets. For example, Bajari and Benkard (2005) study demand for computers where there are many similar products available and it is often possible for consumers to switch to a nearly identical product. However, other contexts with thin markets and significant search costs could be different.

My analysis relies on distributional and functional form assumptions commonly used in the empirical literature and holds these assumptions fixed throughout the paper. However, the main insights can be readily generalized to other settings. For example, I use a discrete choice Logit model with Type I extreme value Logit errors and random coefficients utility. But, the key insight in Result 1 (that we can define the hedonic gradient implicitly using share functions) can be applied to probit models or discrete choice models with no Logit error terms (Berry and Pakes, 2007). The benefit of the Logit model is that the partial derivatives of the share function are easy to derive. But, the same insight can be applied by simulating empirical derivatives of share functions in other discrete choice models, as long as these models have well-defined choice probabilities and share functions.

It would also be interesting to explore the supply side and investigate how changes in the supply of characteristics change the relationship between MWTP for the average versus marginal individual. For example, as supply becomes constrained (for example, when fewer houses with large square footage are built), the marginal individual's MWTP for square footage is likely to increase but the MWTP for the average individual may not change (Bayer et al., 2007).

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